Dynamics of Strategic Manipulation in Ad-Words Auction

Tian-Ming Bu*
Department of Computer Science and Engineering
Fudan University
Shanghai, 200433, P.R.China
031021063@fudan.edu.cn

Xiaotie Deng
Department of Computer Science
City University of Hong Kong
Hong Kong SAR
csdeng@cityu.edu.hk

Qi Qi
Department of Computer Science
City University of Hong Kong
Hong Kong SAR
qi.qi@student.cityu.edu.hk

ABSTRACT

The rise of Internet advertisement has created a demand for new auction models. The ad-words auction, used by Google for this purpose, called the position auction by Varian [15], assigns advertisement slots to top price bidders in a decreasing order of the attractiveness to the viewers, and charge each winning agent, for every click generated from its web-link at the allocated slot, the price of the next lower bid. It has been known that this protocol is not incentive compatible but a pure Nash equilibrium exists for the game among the bidders. Moreover, refined solution concepts based on Nash equilibrium have been proposed for better understanding this game [9, 15]. As the participating agents may not bid their private values in the game, it is not clear whether those existential equilibria can be reached.

In this paper, we are interested in the dynamic process how bidders interact to reach an equilibrium.

We first propose a new solution concept, the *forward-looking Nash Equilibrium*, for the position auction by considering the strategic manipulations of an agent that take into consideration the effect of the existing strategies of other agents, as well as their future responses to its own benefit.

We prove that the forward-looking Nash equilibrium has a unique solution. We note that, in a Nash equilibrium, bidders with higher private values may be stuck with an allocation worse than others with a lower private value of the allocation. The forward-looking Nash equilibrium, on the other hand, guarantees a concept called *output truthful* in that the higher a bidder's true value is, the better allocation it will obtain.

Interestingly, we prove that the forward-looking Nash equilibrium in its pricing and allocation scheme is equivalent to the VCG auction outcome, which was regarded not suitable as too complicated to transfer into it from the current Google's position auction pricing system. In fact, our results justify the use of Google's position auction pricing scheme. In presence of the most sophisticated users, it is indeed equivalent to the Vickrey auction pricing scheme.

Most importantly, we justify the new solution concept by deriving its convergence property. We study several dynamic adjustment schemes by the bidders, including one that converges in a finite number of steps. Moreover, we show that a randomized adjustment scheme will converge to the forward-looking Nash equilibrium with probability one. As the randomized scheme is most reasonable to characterize a situation of uncoordinated bidders, it shows robustness of the forward-looking Nash equilibrium.

*The work was done while the author was at City University of Hong Kong. Research is supported by a research grant of National Natural Science Foundation of China (No. 60496321)

WWW2007, May 8–12, 2007, Banff, Canada.
.

Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

General Terms

Economics, Theory

Keywords

ad-words auction, strategic manipulation, forward-looking Nash equilibrium, dynamics, convergence

1. INTRODUCTION

Internet advertisement market models, where buyers bid for clicks on sponsored links on the Web display space, have attracted intensive research activities in this important emerging market of billions of dollars [1]. The market is usually centered at a keyword which a user submitted to a search engine. In addition to information provided by the search engine, sponsored links are displayed in the hope to tempt the user to click it to the sponsors' own web-pages, which may transfer into sales of their products. The cost of the advertisers is commonly based on the clicks into their advertisement web-pages from the search engine's display of the keyword.

The charging scheme, commonly referred to as "pay-per-click" is designed in a way such that the advertisers will not have to pay if their sponsored links are not clicked upon by the viewer. Google and Yahoo! have adopted a discriminative charging scheme on clicks to displayed links of the advertisers. It can be best described by the *Position Auction*, defined by Varian [15]. Under this model, there are K positions for sale to the advertisers to display their web-links. Each position is associated with a frequency a viewer may click into the web-link placed at that position. The frequency can be regarded as the expected number of clicks on web-links placed at those positions. We sort them in a linear non-increasing order. Intuitively, web-links allocated at higher ranked position will generate more clicks and thus more revenues. Therefore, they will be allocated to those bring in more revenue on the sponsored links and be asked to pay more than those at lower ranked positions. The position auction allocates the K total positions to the K highest price bidders. It charges a click on a higher ranking positioned web-link a price equal to the bidding price of the advertiser placed at the next lower ranked position. The lowest positioned web-link owner pays, for each click on its web-link, the highest bidding price of the losers, i.e., those bidders not allocated a place.

Advertisers have an estimation of the potential value to their products of the users who are interested in the keyword. The estimation is in general different for different advertisers. The difference may result from various reasons, such as the differences in their products, or differences in their marketing strategies. The value is private to the advertisers. They may or may not want to reveal their private values of the click on the keyword. The utility of an advertiser is $c \times (v - p)$, dependent on its private value v, and the position assigned to it in two parameter: the frequency c that the position will be clicked upon, and the price p which is charged on a click of the position. It has been known that the protocol is not incentive compatible but pure Nash equilibrium exists for the game under the protocol [9, 15, 2].

However, a weakness that Nash equilibria as a solution concept in the position auction model is that the bidders may not be willing to bid their true values. This is not a problem when a Nash equilibrium is reached. Each loser knows it is at its optimum allocation losing the game as all the winning prices are at least as high as its own private value. Winners can check, if other agents' biddings are fixed, whether they can be better off by changing their own bidding prices. But how can it be reached? Furthermore, without consideration of the convergence process, the agents may stuck with a Nash equilibrium that is not appropriate.

For example, consider three bidders for two positions. The frequencies for the two positions are 20 and 10. The values are 5, 4 and 1. The set of bids, $b^1 = \$2$, $b^2 = \$4$ and $b^3 = \$1$, result in a Nash equilibrium where agent 2 is assigned the higher position with price 2 and agent 1 the lower position with price 1! Agent 1's utility is $u^1 = (5-1) \times 10 = 40$. But if she bade a value higher than 4 while other bids are fixed, her position would be switched to the higher slot and her utility would be reduced to $u^1 = (5-4) \times 20 = 20$. For the same reason, if Agent 2 bade a value lower than 2, her utility would be at most $(4-1) \times 10 = 30$, which is lower than her utility $(4-2) \times 20 = 40$ in the current allocations and prices. Clearly, Agent 3 also has no incentive to change its own bid. Therefore, this is indeed a pure Nash equilibrium.

Our study is motivated in the dynamic process where an agent may reasonably design its own bidding strategies to improve its utilities. Our approach allows a discovery of a robustness aspect of pure Nash equilibria with respect to potential strategic behaviors of bidders. In Nash equilibria, an agent will choose a strategy that maximizes its own utility with respect to a given set of strategies of other players. It is assumed that they will stay with the incumbent if no other choices will gain it better utility. In arguing for a more robust solution, the agents are assumed to wise enough to explore potential improvement without the possibility of making its payoff reduced.

As in the above example, if Agent 1 became a little smarter, she would by no means keep still in the above equilibrium. For example, if Agent 1 increases her bid to \$2.8, Agent 2's utility will be decreased to $(4 - 2.8) \times 20 = 24$. However, if Agent 2 abandoned the higher slot and selected the lower slot, her utility would be $(4 - 1) \times 10 = 30$. Subsequently, Agent 2 would choose to abandon the higher slot. This solution is also a Nash equilibrium. Further, Agent 1's utility will be at least $(5 - 2.8) \times 20 = 44$, which is larger than that in the above myopic Nash equilibrium. We shall show later that there is a range above the current bid of an agent, within which it can change to any other bid with the possibility that its utility will be increased (and without the possibility of a reduction on its utility) because of the next high bidder may change its bid to a lower bid for its own benefit.

Coincidentally, one unique feature of the online ad-words auction, much due to the convenience provided by computerized processes, is that the advertisers can change their bids anytime. Such flexibility may give advertisers, in order to obtain higher display rank or decrease their charged price or both, an incentive to adjust their bids dynamically. Therefore, such strategic manipulation is possible especially for the application of position auction in online advertisement. While Nash equilibria characterize individual rationality in one-shot game well, the dynamic environment of online auction clearly encourages participating agents to look into the effect of their strategies on future responses of others.

We formalize such a possibility by introducing a new solution concept, the forward-looking Nash Equilibrium. In comparison, we shall use the myopic Nash equilibrium in place for the traditional one-shot Nash equilibrium. In a forward-looking Nash equilibrium, an agent, in choices of its own strategies, takes into consideration the effect of the existing strategies of other agents, as well as their future responses. In myopic Nash equilibria only the effect of existing strategies of other agents are taken into account.

As we have seen in the above example, it is possible that in the myopic Nash equilibrium, bidders with higher private values may stuck with an allocation worse than others with a lower private values of the allocation. We demonstrate the superiority of the new concepts by showing that, the forward-looking Nash equilibrium, on the other hand, guarantees a concept called *output truthful* [11, 6] in that the higher a bidder's true value is, the better allocation it will obtain (and pays more for each click).

Surprisingly, we show that such a dynamic solution results in the same allocations and prices for the bidders as the celebrated VCG mechanism. For the VCG mechanism, the allocation is one of the optimal among all feasible allocations. An agent in the VCG allocation is charged with the difference of the social utility of the optimal allocation without its participation minus the total VCG optimal allocation. The VCG mechanism is incentive compatible but is not applied to the ad-words auction as it is regarded too complicated to the users.

We cannot find any official document explaining why the VCG mechanism is not adopted by the search engine companies. However, compared with the current position auction, obviously the pricing principal used by VCG is more complicated to the advertisers. Further, in VCG, each advertiser's payment is decided by all the others' bids, which makes the advertiser difficult in precisely estimating the payment and control the budget.

The study of a solution concept for dynamic manipulations of participating agents would not be complete without a study on convergence of the dynamic process. Indeed, convergence is a nontrivial matter here. We first present two deterministic ones, obviously fair to all participants, and show that they do not always converge. Then we develop a solution which converge in a finite number of re-adjustments. Finally, we justify the robustness of convergence by proving that randomly selecting an agent to re-adjust will lead to convergence with probability one.

1.1 Related Work

[9, 15, 2, 13] analyze the auction model in terms of equilibria solutions. They all find current ad-words auction models used by Google and Yahoo! are not truthful. In [2], Aggarwal et al. design a *truthful* auction, named *laddered auction*, for pricing keywords. This laddered auction is essentially similar to the VCG mechanisms [16, 7, 10] when the click-through rates is separable. In [9], Edelman et al. regard the ad-words auction as a static one-shot complete information game. Then they study *locally envy-free* equilibria, where no player can improve her payoff by exchanging bids with the player ranked one position above him, which is motivated by some concepts of revenge. It is not difficult to prove that LEFE is a subset of Nash equilibria. Varian's paper [15] studies the Nash equilibrium of the ad-words auction and the relevant properties. Particularly, in consideration of mathematical difficulties, [15] focuses a subset of Nash equilibria, called *Symmetric Nash Equilibria*, which can be formulated nicely and dealt with easily. In [13], Lahaie studied two slot auction mechanisms: rank by bid and rank by revenue used currently by Yahoo! and Google respectively.

It has been proved that both the revenues under a locally envy-free equilibrium and a symmetric Nash equilibrium return to the auctioneer *at least* the same revenue as that under the VCG mechanism ([9] and [15]). Such a property justifies the use of the position auction, to a theoretician's satisfaction that the sacrifice of incentive compatibility in preferring the position auction to the VCG auction potentially provides the auctioneer with some extra benefit.

Actually, this property was not accidentally satisfied by both solution concepts. They are actually the same concept expressed in different forms. Furthermore, the revenue under our unique forward-looking Nash equilibrium is the same as the lower bound under Varian's symmetric Nash equilibria [15] and the lower bound under Edelman et al.'s locally envy-free equilibria.

Comparatively less is regarded this model as a dynamic system and studies bidders' strategic behavior and the stability of the system. Edelman et al. [8] show the instability of first-price auction by Overture's bidding data. The "saw-tooth" pattern presents the oscillation in the system. Both [12] and [4] study bidders' behavior of bidding several keywords with budget constraint. In [12], Kitts et al. design an intelligent trading agent for ad-words auctions. In [4], Borgs et al. present an optimal strategy for bidders based on the heuristic that each bidder should bid an amount such that his "return-on-investigation" is equal across all keywords. They also discuss the stability of this heuristic. Asdemir [3] considers the ad-words auction as alternative-move game and studied this dynamic model with only two symmetric bidders based on the strategies which only depend on the payoff relevant history. Another idea to investigate bidders behavior is from the interesting concept "antisocial behavior" proposed by Brandt and Weiß [5]. In this concept, the bidder's behavior is directed by the tradeoff of increment of the bidder's payoff and the decrement of the competitors' payoff, instead of just maximizing the bidder's payoff. [12, 14, 17] all consider this issue in the ad-words auction. Specially, in [17], Zhou et al. study the equilibria under "vindictive bidding" which means one bidder forces his competitor to pay more without affecting his own payment. In other words, the bidder's bid is one cent below his competitor's bid.

This paper is organized as follows. Section 2 presents the position auction model and related notations. In section 3, we discuss agent strategic manipulations and study the best strategic manipulation function of the agents. We prove that our new solution concept to reflect their strategic manipulations results in the same allocations and the same pricing scheme as the VCG mechanism. Therefore, it unites the two important methodologies for market pricing processes, Nash equilibrium (in a stronger sense) and incentive compatibility. In section 4, we investigate the convergence properties toward forward-looking Nash equilibrium of participating agents in the position auction. Finally, we conclude in Section 5 with discussion on future research issues.

2. PRELIMINARIES

The position auction protocol (Varian [15]) models after keyword advertisement auctions adopted by Google and Yahoo!.

For some keyword, consider a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of advertisers who bid for $\mathcal{K} = \{1, 2, \ldots, K\}$ advertisement slots ($K < N$) which will be displayed alongside with the search result page. Usually, a well positioned advertisement would receive more clicks than poor position ones. We should index them in a decreasing order of the popularity of those slots, i.e., $c_1 > c_2 > \cdots > c_K$. Therefore, for any two slots $k_1, k_2 \in \mathcal{K}$, if $k_1 < k_2$, then slot k_1's click-through-rate (CTR for short) c_{k_1} is larger than c_{k_2}. We reasonably assume $c_K > 0$. Moreover, each bidder $i \in \mathcal{N}$ has a privately known information, v^i, which represents the expected return of per-click to bidder i. For simplicity, it is assumed that all the bidders' private values are different.

Given a set of bids submitted, bid b^i from Advertiser i, $1 \leq i \leq N$, the auctioneer faces a decision on how to distribute the advertisement slots among the bidders and how much each should pay for a click. The position auction assigns the highest slot to the highest bidder, the second highest slot would be allocated to the second highest bidder, and so on. The last $N - K$ bidders would lose and get nothing. Finally, each winner would be charged for per-click the next bid to him in the descending bid queue. The losers would pay nothing. We also assume all the bids would be always different.

Let b_k denote k^{th} highest bid in the descending bid queue and v_k the true value of the k^{th} bidder in the descending queue. So if Bidder i got slot k, i's payment would be $b_{k+1} \cdot c_k$. Otherwise, his payment would be zero. Hence, for any bidder $i \in \mathcal{N}$, if i were on slot $k \in \mathcal{K}$, his utility (payoff) could be represented as

$$u_k^i = (v^i - b_{k+1}) \cdot c_k \tag{1}$$

Different from one-round sealed-bid auctions where each agent has only one chance to bid, ad-words auction allows changes of their bids anytime. Once some bid were changed, the system would refresh the ranking automatically and instantaneously. Accordingly, all the bidders' payment and utility would also be recalculated. That leaves a room for an agent to make strategic manipulations to force other agents to change their bids.

We remark that all the bidders are selfish and may not report their private values truthfully. What the bidder would consider and only considers is how much he should bid to maximize his utility.

In [9] and [15], two new solution concepts were introduced to amend Nash Equilibria with additional, but quite different, rationality arguments.

Definition 1. [15] A Symmetric Nash Equilibrium (SNE) is a set of prices such that

$$(v_s - p_s)x_s \geq (v_s - p_t)x_t \text{ for all } t \text{ and } s.$$

Definition 2. [9] An equilibrium of the static game induced by GSP is locally envy-free if a player cannot improve his payoff by exchanging bids with the player ranked one position above him.

Even though the two concepts are motivated quite differently, they are in fact defining the same set as known by the research community of this field.

THEOREM 1. *(Folklore) The set of symmetric Nash equilibria (SNE) is equal to the set of locally envy-free equilibria (LEFE)*

3. FORWARD-LOOKING NASH EQUILIBRIUM FOR POSITION AUCTION

In this Section, we illustrate a type of strategic manipulations that agent can exercise to explore the possibility of profiting by forcing other agents to abandon their current bids to a lower position. Such strategic behavior points to the vulnerability of Nash equilibrium in position auctions and calls for a more comprehensive understanding of the equilibrium under the circumstance of online advertisement with repeated biddings, where agents have ample opportunity for exploiting any weakness. While we share the same motivation as the work of that introduced SNE and LEFE [15, 9], our approach is different. The previous definitions are based on reasoning on fairness. Ours studies the dynamic outcome of rational strategic manipulations by participating agents.

3.1 Exploitable Vulnerability of Nash Equilibria

Since each bidder's utility function depends on the bid of the bidder on the next slot, it turns out that the optimal decision of each bidder depends on the decisions made by other bidders. Symmetrically, since each bidder's decision would also influence the other bidders' profit, his decision would also influence the other bidders' decisions in future.

However, traditional Nash equilibria assume that each bidder is myopic. In other words, unless the bidder could benefit immediately from changing her bid, she would keep still. The bidder would never consider those promising decisions which not only wouldn't decrease her profit, but also would benefit her indirectly by influencing the other bidders' decisions firstly. Unfortunately, this weakness in assumption is blown up in the ad-words auction game. This neglected incentive plays an important role in the game and the traditional Nash equilibrium may become unstable in practice.

Consider the example in the introduction. There are a total of two slots, $c_1 = 20$ and $c_2 = 10$, and three bidders $i = 1, 2, 3$, whose private true value is $v^1 = 5$, $v^2 = 4$ and $v^3 = 1$. As we have seen, the collection of bids, $b^1 = \$2$, $b^2 = \$4$ and $b^3 = \$1$ is a myopic Nash equilibrium. If bidder 1 became a little smarter, she would by no means keep still in the above equilibrium. Since bidder 2's utility in the equilibrium depends on bidder 1's bid, bidder 1 would increase her bid so high as to compel bidder 2 to abandon the higher slot, instead of bidding a value higher than 4 to obtain c_1 directly. On the other hand, since bidder 1's payoff may depend on bidder 2's bid in the next step, she would not infinitely raise her bid. To clarify the idea, we argue that bidder 1 may adopt the following practical strategy, starting from the above myopic Nash equilibrium. In conformity with the practical rules, we assume that the difference between any two bids is at least \$0.01.

Strategy 1 The more advisable strategy for bidder 1

$b^1 = \$2$
while bidder 2 wouldn't like to abandon the higher slot **do**
  $b' = b^1 + \$0.01$
  //Check whether bidder 1's utility would decrease, in case bidder 2 abandoned the higher slot and bade a bid very close to bidder 1's bid.
  if $(5 - (b' - 0.01)) \times 20 < (5 - 1) \times 10$ **then**
    Keep still and jump out the loop
  else
    Submit a new bid b' and let $b^1 = b'$
  end if
end while

According to the strategy, bidder 2 wouldn't abandon the higher slot until bidder 1 increases his bid to \$2.51. Clearly, bidder 1's utility will be at least $(5 - 2.5) \times 20 = 50$, which is one fourth larger than that in the initial myopic stable status.

This property is not limited to this example at all. Bidder 1 can safely adopt the following strategy without possibility of lower utility and with the possibility of better utility.

Strategy 2 The general strategy for ad-words auctions

1: For each bidder, determine her bid range which could maximize her utility in the current situation;
2: In the above optimal range, select a bid so that she could obtain the slot as high as possible after the other bidders' responses for her bid;
3: Be sure that her utility would not be decreased if she really got the higher slot in the next step.

3.2 Forward-looking best-response function and its properties

In the previous section, we have shown the vulnerability of Nash equilibria and have discovered some ideas about a better solution concept. In this subsection, we will present the formal definition of a forward-looking best response function based on the above farseeing strategy and discuss its suitability for characterizing ad-words auctions.

Let $\mathbf{b}$ represent the bid vector $(b^1, b^2, \ldots, b^N)$. $\forall i \in \mathcal{N}$, we denote by $\mathcal{O}^i(\mathbf{b})$ bidder i's place in the descending bid queue. Let $\mathbf{b}^{-i} = (b^1, \ldots, b^{i-1}, b^{i+1}, \ldots, b^N)$ denote the bids of all other bidders except i.

Definition 3. **(Myopic Best-response Function)** Given $\mathbf{b}^{-i}$, bidder i's myopic best-response function $\mathcal{M}^i(\mathbf{b}^{-i})$ returns a set defined as

$$\mathcal{M}^i(\mathbf{b}^{-i}) = \arg\max_{b^i \in [0, v^i]} \left\{ u^i_{\mathcal{O}^i(b^i, \mathbf{b}^{-i})} \right\} \tag{2}$$

Clearly, eventually all the losers would submit their true values to enhance the possibility of winning the last slot, as they bid their ways toward a possibility of winning a slot of price lower than their true values without success.

With the other agents' bids $\mathbf{b}^{-i}$ fixed, consider $\mathcal{O}^i(\mathcal{M}^{-i}(\mathbf{b}^{-i}, b^i), b^i)$ as a single-variable function of b^i.

PROPOSITION 1. *For any agent i, $\mathcal{O}^i(\mathcal{M}^{-i}(\mathbf{b}^{-i}, b^i), b^i)$ is monotone non-increasing in terms of b^i.*

PROOF. Suppose $b^i < \tilde{b}^i$, $\mathcal{O}^i(\mathbf{b}^{-i}, b^i) = k$. For some bidder $j \in \mathcal{N}$, $j \neq i$, if bidder j prefers a slot t lower than k after i bids b^i, obviously j must still prefer the slot t if i bids $\tilde{b}^i$ instead of b^i initially.

So after bidder i increases her bidder from b^i to $\tilde{b}^i$, the bidders who prefer slots lower than k would still prefer these slots. Hence bidder i will eventually get a slot when she bids $\tilde{b}^i$ at least as high as k when she bids b^i.

Therefore, if $b^i < \tilde{b}^i$,

$$\mathcal{O}^i(\mathcal{M}^{-i}(\mathbf{b}^{-i}, b^i), b^i) \geq \mathcal{O}^i(\mathcal{M}^{-i}(\mathbf{b}^{-i}, \tilde{b}^i), \tilde{b}^i)$$

. $\square$

Given $\mathbf{b}^{-i}$, $\mathbf{b}^i \in \mathcal{M}^i(\mathbf{b}^{-i})$. Suppose $\mathcal{O}^i(b^i, \mathbf{b}^{-i}) = k$, the bidder on slot $k+1$ bids b_{k+1}, and $\mathcal{O}^i(\mathcal{M}^{-i}(\mathbf{b}^{-i}, b^i), b^i) = t$, then we have

PROPOSITION 2. *If $u^i_k \leq u^i_t$ for all $t : t < k$, then*

$$b^i \leq v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1})$$

PROOF. When bidder i gets slot k, her utility is $u^i_k = (v^i - b_{k+1})c_k$. When bidder i gets slot t, her utility is

$$\begin{aligned} u^i_t =& (v^i - b_t)c_t \\ \geq& (v^i - b^i)c_t \end{aligned}$$

From $\forall t < k,\ u^i_k \leq u^i_t$, we have

$$\begin{aligned} &(v^i - b_{k+1})c_k \leq (v^i - b^i)c_t \\ \Rightarrow& b^i c_t \leq v_i(c_t - c_k) + b_{k+1}c_k \\ \Rightarrow& b^i \leq v^i - \frac{c_k}{c_t}(v^i - b_{k+1}) \\ &\text{Since } v^i > b_{k+1} \text{ and } \forall t < k, c_t \geq c_{k-1} \\ \Rightarrow& b^i \leq v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1}) \end{aligned}$$

$\square$

From proposition 1, we learn that for any given $\mathbf{b}^{-i}$, the higher bidder i bids, the higher a slot she can get in the next step. Therefore, it implies that any bidder i should bid as high as possible in the set $\mathcal{M}^i(\mathbf{b}^{-i})$ in order to get a higher slot in the next step. However, according to proposition 2, subject to no risks of the decrease of its own payoff by the affected bidders' next optimal move if she gets the higher slot in the next step, bidder i preferring slot k would not bid higher than $v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1})$. To see it differently, if this bidder gradually increases its bid by a minimum increment, it would stop bidding higher at this point.

If one preferred the highest slot, clearly, she would bid as high as possible so that she could still obtain the highest slot after the other bidders' responses. So she would bid her true value. Similarly, as for the losers, she would her true value in order to get some slot after the other bidders' responses for her bid. So she would also bid her true value. So strategy 2 is exactly the forward-looking best response function defined as follows.

Definition 4. **(Forward-Looking Best Response Function)** Given $\mathbf{b}^{-i}$, suppose $\mathcal{O}^i(\mathcal{M}^i(\mathbf{b}^{-i}), \mathbf{b}^{-i}) = k$, then bidder i's forward-looking response function $\mathcal{F}^i(\mathbf{b}^{-i})$ is defined as

$$\mathcal{F}^i(\mathbf{b}^{-i}) = \begin{cases} v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1}) & 2 \leq k \leq K \\ v^i & k = 1 \text{ or } k > K \end{cases} \tag{3}$$

Then,

Definition 5. **(Forwarding-Looking Equilibria)** A forward-looking best response function based equilibrium is a strategy profile $\hat{\mathbf{b}}$ such that

$$\forall i \in \mathcal{N}, \hat{\mathbf{b}}^i \in \mathcal{F}^i(\hat{\mathbf{b}}^{-i})$$

The following results show the above forward-looking best response function is well defined and has many good properties.

Definition 6. **(Output Truthful)** [11, 6] For any instance of ad-words auction and the corresponding equilibrium set $\mathcal{E}$, if $\forall \mathbf{e} \in \mathcal{E}$ and $\forall i \in \mathcal{N}$, $\mathcal{O}^i(\mathbf{e}) = \mathcal{O}^i(v^1, \ldots, v^N)$, then we say ad-words auction is *output truthful* on $\mathcal{E}$.

THEOREM 2. *Ad-words auction is output truthful on $\mathcal{E}_{forward-looking}$.*

PROOF. Suppose there exists an instance of ad-words auction which is not output truthful in terms of some equilibrium $\mathbf{e}$, then in the equilibrium $\mathbf{e}$, there must exist a pair of adjacent slots k, $k+1$ and the bidder i on slot k and the bidder j on slot $k+1$ such that $v^i < v^j$, which may only occurs when $k < K$.

Since $(v^j - b_{k+2}) > 0$ and $(c_k - c_{k+1}) > 0$, $(v^j - b_{k+2}) \cdot (c_k - c_{k+1}) > 0$, then $\left(v^j - \frac{c_{k+1}}{c_k}(v^j - b_{k+2})\right) > b_{k+2}$.

According to equation (3), bidder i's utility on slot k is

$$\begin{aligned} u^i_k =& (v^i - b_{k+1}) \cdot c_k \\ =& \left(v^i - \left(v^j - \frac{c_{k+1}}{c_k}(v^j - b_{k+2})\right)\right) \cdot c_k \\ =& (v^i - b_{k+2}) \cdot c_{k+1} + (c_k - c_{k+1}) \cdot (v^i - v^j) \\ <& (v^i - b_{k+2}) \cdot c_{k+1} \\ =& u^i_{k+1} \end{aligned}$$

So bidder i should prefer slot $k+1$ than slot k, which contradicts the definition of forward-looking Nash equilibria. $\square$

Corollary 1. *Ad-words auction has an unique forward-looking Nash equilibrium.*

Proof. According to theorem 2, since ad-words auction is output truthful on forward-looking Nash equilibrium, the bid on any slot is fixed. Then combined with equation (3), clearly, there is a unique forward-looking Nash equilibrium. □

Corollary 2. *The ad-words auction is social efficient under the forward-looking Nash equilibrium.*

Furthermore, without loss of generality, we assume that $v^1 > v^2 > \cdots > v^N$.

Corollary 3. *Any bidder's payment under the forward-looking Nash equilibrium is equal to her payment under VCG mechanism for the auction.*

Proof. First, under the VCG mechanism [16, 7, 10], all the bidders would submit their bids truthfully. So VCG mechanism is also output truthful. Clearly, the output under VCG mechanism is the same as the output under ad-words auctions.

Second, according to the definition of VCG mechanism, for any winner $i \le K$, winner i's payment is

$$\begin{aligned}
p^i_{VCG} =& \left(\sum_{i-1}^{k=1} v^k \cdot c_k + \sum_{K+1}^{k=i+1} v^k \cdot c_{k-1} \right) \\
&- \left(\sum_{i-1}^{k=1} v^k \cdot c_k + \sum_{K}^{k=i+1} v^k \cdot c_k \right) \\
=& \sum_{K+1}^{k=i+1} v^k \cdot (c_{k-1} - c_k) \\
=& v^{i+1} \cdot (c_i - c_{i+1}) + p^{i+1}
\end{aligned} \tag{4}$$

In order to describe the above equation more clear, we illustrate its geometric explanation in Figure 1. As in Figure 1, bidder K's VCG payment is exactly the area of rectangle A, bidder $(K-1)$'s VCG payment is both the area of rectangle A and B, bidder $(K-2)$'s VCG payment is all the area of rectangle A, B and C, bidder $(K-3)$'s VCG payment is ...

Now return to check the bidders' payment under the forward-looking Nash equilibrium. For any winner $i \le K$, winner i's payment is

$$\begin{aligned}
p^i_{forward-looking} =& b_{i+1} \cdot c_i \\
=& \left(v^{i+1} - \frac{c_{i+1}}{c_i} (v^{i+1} - b_{i+2}) \right) \cdot c_i \\
=& v^{i+1} \cdot c_i - v^{i+1} \cdot c_{i+1} + b_{i+2} \cdot c_{i+1} \\
=& v^{i+1} \cdot (c_i - c_{i+1}) + p^{i+1}
\end{aligned} \tag{5}$$

Furthermore, $p^K_{forward-looking} = p^K_{VCG} = v^{K+1} \cdot c^K$. Therefore, both equation (4) and (5) are exactly equivalent. So any bidder's payment under the forward-looking Nash equilibrium is equal to her payment under VCG mechanism for the auction. □

Corollary 4. *For ad-words auction, the auctioneer's revenue in the forward-looking Nash equilibrium is equal to her revenue under the VCG auction protocol.*

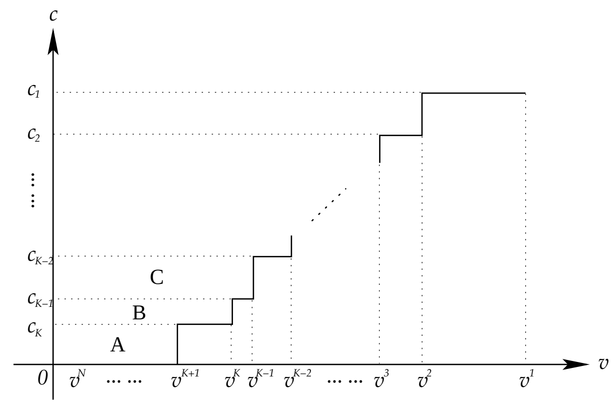

Figure 1: The illustration of how to compute the VCG payment.

4. CONVERGENCE OF AD-WORDS AUCTIONS

At the forward-looking Nash equilibrium, no bidder would gain by deviating from it. In addition, no bidder could suffer an immediate lose by another bidder's move affected by the forward-looking best response. This implies that once the forward-looking Nash equilibrium is reached, the system will remain in that equilibrium.

However, starting from an arbitrary initial state, how and whether selfish players can actually arrive at such an equilibrium is still a problem. In the following parts, we will study several reasonable dynamic models of bidders' behavior for the possibility of convergence to the equilibrium case by case. We show that two apparently fair scheme may not always guarantee convergence to the forward-looking Nash equilibrium. On the other hand, we show that, the convergence toward the forward-looking Nash equilibrium is highly possible in a system of spontaneous bidders.

4.1 Simultaneous Readjustment Scheme

We first consider a practical case where the bidders update their bids simultaneously. I.e, all bidder participating in the auction will use forward-looking best-response function $\mathcal{F}$ to update their current bids simultaneously, which turns the current stage into a new stage. Then based on the new stage, all bidders may update their bids again. Bidders keep updating simultaneously until the reaching of the forward-looking Nash equilibrium.

For each bidder $i \in \mathcal{N}$, denote $b^i(t)$ as the bid value of bidder i at the t^{th} stage, where $t \ge 0$ and $b^i(0)$ is the initial bid value of bidder i. Then $\forall t$, $\forall i$, we have the iteration function:

$$b^i(t+1) = \mathcal{F}^i(\mathbf{b}^{-i}(t)) \tag{6}$$

A crucial question now is the dynamics of reaching that equilibrium, namely find a finite t such that $b^i(t+1) = b^i(t)$ for $\forall i \in \mathcal{N}$. The following counter example shows that the equilibrium may never be reached.

Counter example:

Several bidders compete for 3 slots. The click through rate of each slot is given in table 1(a). Since all bidders use

(a) The click-through rate

slot	1	2	3
CTR	16	11	10

(b) The evolution of bids

true value	$\mathbf{b}(t)$	$\mathbf{b}(t+1)$	$\mathbf{b}(t+2)$
5	1.36	5	1.36
4.8	1.35	4.8	1.35
4.5	1.32	4.5	1.32
1	1	1	1

Table 1: The counter example for the simultaneous readjustment scheme

the forward-looking best-response function to readjust their bids. It's easy to find that after finite steps, the bidders who don't get any slot will bid their true values and from then on their bids will never be changed. Thus, for simplicity, we only consider the updates of the four bidders with highest true values.

Assume the four bidders are $i = 1, 2, 3, 4$, whose private true value is $v^1 = 5$, $v^2 = 4.8$, $v^3 = 4.5$, $v^4 = 1$. At stage t, they bid $b^1(t) = 1.36$, $b^2(t) = 1.35$, $b^3(t) = 1.32$, $b^4(t) = 1$, and $\mathbf{b}(t) = (1.36\ 1.35\ 1.32\ 1)^T$. Since bidder 4 is a loser, she always bids her true value.

Based on the stage t, each bidder updates their bids simultaneously.

From equation 3 and equation 6,

$$b^1(t+1) = \mathcal{F}^1(b^2(t), b^3(t), b^4(t)) = 5$$
$$b^2(t+1) = \mathcal{F}^2(b^1(t), b^3(t), b^4(t)) = 4.8$$
$$b^3(t+1) = \mathcal{F}^3(b^1(t), b^2(t), b^4(t)) = 4.5$$
$$b^4(t+1) = \mathcal{F}^4(b^1(t), b^2(t), b^3(t)) = 1$$
$$b^1(t+2) = \mathcal{F}^1(b^2(t+1), b^3(t+1), b^4(t+1)) = 1.36$$
$$b^2(t+2) = \mathcal{F}^2(b^1(t+1), b^3(t+1), b^4(t+1)) = 1.35$$
$$b^3(t+2) = \mathcal{F}^3(b^1(t+1), b^2(t+1), b^4(t+1)) = 1.32$$
$$b^4(t+2) = \mathcal{F}^4(b^1(t+1), b^2(t+1), b^3(t+1)) = 1$$

So $\mathbf{b}(t+2) = (1.36\ 1.35\ 1.32\ 1)^T$.

Since $\mathbf{b}(t) = \mathbf{b}(t+2)$, which implies that bidders will keep jumping between this two stages time and again, and never reach the equilibrium. Therefore, we have the following proposition:

PROPOSITION 3. *Ad-words auction may not always converge to forward-looking Nash equilibrium under the simultaneous readjustment scheme even when the number of slots is 3.*

We comment that 3 is the minimum number of slots when this strategy fails as it is not hard to prove that the protocol always converges when the number of slots is 2.

The main steps of this example are shown in table 1(b).

4.2 Round-Robin Readjustment Scheme

In this subsection, we consider another scheme called the round-robin readjustment scheme, which is popular because of its fairness and operability. Under this scheme, bidders take turns to update one after another, i.e, there doesn't exist two bidders who update simultaneously. Here we divide this scheme into two types according to the two elements in the auction: bidder and slot.

(a) The click-through rate

slot	1	2	3	4
CTR	74	68	41	25

(b) The evolution of bids

No.	true value	$\mathbf{b}(t)$	$\mathbf{b}(t+1)$	$\mathbf{b}(t+2)$
1	305.14	174.75	155.87	174.75
2	209.41	142.69	160.21	142.69
3	197.67	138.12	163.24	138.12
4	797.51	797.51	797.51	797.51
5	100.00	100.00	100.00	100.00

Table 2: The counter example for the readjustment according to bidder number

4.2.1 *Readjustment according to bidders*

Consider bidders update their bids sequentially, in the order $1, 2, \ldots, N, 1, 2, \ldots, N$, etc, where $1, 2, \ldots, N$ is bidder number randomly allocated to each bidder. Now for $\forall i$, $\forall t$, the iteration function is:

$$b^i(t+1) = \mathcal{F}^i(b^1(t+1), \ldots, b^{i-1}(t+1), b^{i+1}(t), \ldots, b^N(t)) \tag{7}$$

In order to show the convergence, we need to prove that for any given bid set $\mathbf{b}$, there exists a t, such that for $\forall i$, $b^i(t+1) = b^i(t)$. However, this time we still find a counter example which shows that the equilibrium may never be reached.

Counter example:

5 bidders competes for 4 slots. Randomly sort these five bidders and give each one a bidder number according to the order. The click through rate of each slot is given in table 2(a) and the first two columns of table 2(b) give the bidder number and the corresponding private true values. The remaining columns of table shows the iteration processes. Here we only demonstrate the iteration process of bidder 2 in details and the other bidders' processes are similar to bidder 2's.

From equation 7,

$$\begin{aligned} b^2(t+1) =& \mathcal{F}^2(b^1(t+1), b^3(t), b^4(t), b^5(t) \\ =& \mathcal{F}(155.87, 138.12, 797.51, 100.00) \\ =& 160.21 \\ b^2(t+2) =& \mathcal{F}^2(b^1(t+2), b^3(t+1), b^4(t+1), b^5(t+1) \\ =& \mathcal{F}(174.75, 163.24, 797.51, 100.00) \\ =& 142.69 \end{aligned}$$

Since $b^i(t) = b^i(t+2)$ for all i and $b^i(t) \neq b^i(t+1)$ for some i, readjustment according to bidder number may not convergent to forward-looking Nash equilibrium.

4.2.2 *Readjustment according to slots*

We create $N - K$ virtual slots with click through rate $c_{K+1} = c_{K+2} = \cdots = c_N = 0$, thus the bidders and slots have one-to-one relationship.The order of slots is equivalent to the order of bids. Then we consider that bidders update their strategies sequentially according to the order of slots.

(a) The click-through rate

slot	1	2	3	4	5
CTR	74	68	41	25	0

(b) The evolution of bids

true value	$\mathbf{b}(t)$	$\mathbf{b}(t+1)$	$\mathbf{b}(t+2)$	$\mathbf{b}(t+3)$
797.51	797.51	797.51	797.51	797.51
305.14	178.18	155.86	178.18	178.18
209.41	142.69	160.20	166.98	142.69
197.67	138.12	163.24	163.24	138.12
100.00	100.00	100.00	100.00	100.00

Table 3: The counter example for the readjustment according to slot (top down)

Next, we consider the two cases of this adjustment: ascending case and descending case.

Case I: descending case (top down)

Bidders with higher slot update their bids first. Now the iteration function is:

$$b_i(t+1) = \mathcal{F}(b_1(t+1), ..., b_{i-1}(t+1), b_{i+1}(t), ..., b_N(t)) \quad (8)$$

Unfortunately, we still find a non-convergence example under this situation.

Counter example (top down):

5 bidders compete for 5 slots. The click through rate of each slot is given in table 3(a). Table 3(b) gives the bidders' private true values and the main steps of the dynamics process.

Here we only give the iteration process of the bidder with true value of 209.41.

From equation 8,

At stage t, she gets slot 3. At stage $t+1$, she updates her bid to

$$\begin{aligned} b_3(t+1) =& \mathcal{F}(b_1(t+1), b_2(t+1), b_4(t), b_5(t)) \\ =& \mathcal{F}(797.51, 155.86, 138.12, 100.00) \\ =& 160.20 \end{aligned}$$

After the remaining bidders update, she still obtains slot 3 at stage $t+1$. Then at stage $t+2$, she updates her bid to

$$\begin{aligned} b_3(t+2) =& \mathcal{F}(797.51, 163.24, 155.86, 100.00) \\ =& 166.98 \end{aligned}$$

This time she gets slot 3 and at stage $t+3$, she updates her bid to

$$\begin{aligned} b_3(t+3) =& \mathcal{F}(797.51, 178.18, 163.24, 100.00) \\ =& 142.69 \end{aligned}$$

Since $b_i(t) = b_i(t+3)$ for $\forall i$ and $b_i(t) \neq b_i(t+1)$, $b_i(t) \neq b_i(t+2)$ for some i, we have the conclusion that forward-looking equilibrium is not convergent under this readjustment scheme.

Case II: ascending case (bottom up)

Bidders with lower slot update their bids first. Then the iteration function is:

$$b_i(t+1) = \mathcal{F}(b_1(t), ..., b_{i-1}(t), b_{i+1}(t+1), ..., b_N(t+1)) \quad (9)$$

Still we find a non-convergence example under this situation.

(a) The click-through rate

slot	1	2	3	4	5
CTR	96	75	41	15	0

(b) The evolution of bids

true value	$\mathbf{b}(t)$	$\mathbf{b}(t+1)$	$\mathbf{b}(t+2)$
100.00	100.00	100.00	100.00
827.18	561.14	686.00	561.14
828.6	562.04	562.04	562.04
948.01	646.47	743.31	646.47
1055.10	1055.10	1055.10	1055.10

Table 4: The counter example for the readjustment according to slot (bottom up)

Counter example (bottom up):

5 bidders compete for 5 slot. Table 4(a) gives the click through rate of each slot. Bidders' private true values and the dynamics process. The second and the fourth column of table 4(b) have the same values, which implies that bidders can not dynamically reach the forward-looking Nash equilibrium.

Now we can get the following proposition:

PROPOSITION 4. *Ad-words auction may not always converge to forward-looking Nash equilibrium under the round-robin readjustment scheme even when the number of slots is 4.*

We comment that 4 is the minimum number of slots when this strategy fails as it is not hard to prove that the protocol converges when the number of slots is 3.

4.3 Randomized Readjustment Scheme

In reality, there is no centralized controller who decides the priority based on some adjustment scheme in on-line ad-words auctions. The decisions are made spontaneously by individual bidders. They are no longer deterministic in advance, but are at random.

It is therefore important to study the case where the bidders update their bids in a randomized manner. We formally model such a behavior pattern by randomly pick a bidder to change its decision. We still assume that all bidder participating in the auction will use the forward-looking best-response function $\mathcal{F}$ to update their current bids.

First, by constructing a special deterministic adjustment rule, we prove that the ad-words auction will reach the forward-looking equilibrium in finite steps under the condition that there are no two bidders update simultaneously. Then we prove the convergence property in the randomized adjustment scheme.

We construct a special updating rule for a K slot ad-words auction with N bidders ($K < N$) called Lowest-First as Strategy 3 as below. We recall that the current bids are ordered (and assumed distinct) in $b_1 > b_2 > \cdots > b_j > \cdots > b_N$. We use index j be the bid for which we will update according to our best-response function.

LEMMA 1. *Ad-words auction converge to forward-looking Nash equilibrium in finite steps with the strategy Lowest-First.*

PROOF. We prove it by induction. First of all, $N = 1$ is trivial as $K = 0$. Consider the case $N = 2$ and $K = 1$.

Strategy 3 Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$

if $(j = 0)$ **then**
  exit
end if
Let i be the ID of the bidder whose current bid is b_j (and equivalently, b^i).
Let $h = \mathcal{O}^i(\mathcal{M}^i(\mathbf{b}^{-i}), \mathbf{b}^{-i})$.
Let $\mathcal{F}^i(\mathbf{b}^{-i})$ be the best response function value for Bidder i.
Re-sort the bid sequence. (So h is the slot of the new bid $\mathcal{F}^i(\mathbf{b}^{-i})$ of Bidder i.)
if $(h < j)$ **then**
  call Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$,
else
  call Lowest-First$(K, h-1, b_1, b_2, \cdots, b_N)$
end if

When the procedure is called at Lowest-First$(1, 2, b_1, b_2)$ as $K = 1$ and $N = 2$, the bidder $j = 2$ with bid b_2 would choose its best response with respect to b_1.

If its best response is less than b_1, then the new bid will be its own private value according to the best response function as its is a loser after the bid. The recursive call will be Lowest-First$(1, 1, b_1, b_2)$ and the bidder $j = 1$ will also bid its own private value according to the best response function.

Otherwise, if the bidder $j = 2$'s best response is large than b_1, then the bid will again be its own private value since it is the highest bid. Then the recursive call will be again Lowest-First$(1, 2, b_1, b_2)$ but it is further constrained that b_1 is the true private value of the bidder $j = 1$ (note that the index is updated here). Then, when bidder $j = 2$ bids in the recursive call, it is always its true value as in the above analysis. Then, both bids will be the true private values of the bidders and the procedure will terminate as we go through the procedures.

In general, Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ is first called with $j = N$. It needs a proof that Lowest-First$(K, j, b_1, b_2, \cdots \cdots, b_N)$ with $j = 0$ will be eventually called and the procedure stops within a finite number of steps. We outline the proof structure as follows: First, we prove that Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ will be call with $j = N - 1$ after a finite number of steps of calls to Lowest-First$(K, j, b_1, b_2, \cdots \cdots, b_N)$ with $j = N$. Second, we prove, a recursive call to Lowest-First$(K, N, b_1, b_2, \cdots, b_N)$ within a call to Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ can occur at most once for the bidder with bid b_j and the bidder with bid b_N. Finally we comment that the same property (by inductive proof) holds for each pair of ordered indices in the recursive calls. It follows that Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ with $j = 0$ will be called. The procedure ends within a finite number of steps.

For the correctness proof, we note when Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ is called, bids $(b_{j+1}, b_{j+2}, \ldots, b_N)$ all obey the best response function. When $j = 0$ is called, all indices obey the best response function and the result is the forward-looking Nash equilibrium.

To finalize the proof, we note that for a consecutive number of recursive calls to Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ with $j = N$, at each call the bidding vector $(b_1, b_2, \cdots, b_N)$ will increase in at least one coordinate. Let the bidder changing its bid has a private value v, the new bid will move closer to v within a ratio α, where $\alpha = \min\{\frac{c_{i+1}}{c_i} : i = 1, 2, \ldots, N-1\}$. Let $\delta = \min\{|v_a - v_b| : a \neq b\}$, the total number of consecutive calls to Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ with $j = N$ will terminate after $N \cdot g$ steps for $g : \forall v : v \cdot \alpha^g < \delta$.

Since all bidders obey forward-looking best-response function, which means after finite steps, the bidders who don't get any slot will bid their true values and from then on their bids will never be changed. Thus, for simplicity, we only consider the updates of the K bidders with highest true values.

A recursive call to Lowest-First$(K, N, b_1, b_2, \cdots, b_N)$ within a call to Lowest-First$(K, j, b_1, b_2, \cdots, b_N)$ can occur only if the bidder with bid b_j has a private value smaller than that of the bidder with bid b_N. Therefore, the private values at b_N is non-increasing. The claim follows.

□

THEOREM 3. *Ad-words auctions converge to forward-looking Nash equilibrium with probability one under randomized readjustment scheme.*

PROOF. Denote by $l = l(c_1, c_2, \cdots, c_K, v_1, v_2, \cdots, v_N)$ be the number of adjustment that guarantees convergence in the Lowest-First strategy.

Under randomized readjustment scheme, each round we randomly choose one bidder who wants to change his bid updating. There is a fixed non-zero probability that the necessary convergence sequence is chosen for a run of l adjustments. The probability will be boosted as the number of runs multiplies, and it follows that the forward-looking Nash equilibrium will be reached in finite number of steps with probability one.

□

5. CONCLUSIONS

In this paper, we regard the ad-words auction as a dynamic incomplete information noncooperative game where every player will take into account both his current behavior and his effect on the other player's future behaviors. This approach surprisingly integrates the concept of Nash equilibrium and the concept of incentive compatibility in proving that they achieve the same outcome.

We also investigate the convergence of the new equilibrium concept under the *simultaneous*, *round-robin* and *randomized* adjustment schemes respectively.

It should be noted that, from section 4.3, the convergence property still exists in the randomized adjustment scheme even if there are several bidders update their strategies concurrently. Actually, this is exactly the reality in on-line adwords auctions - all the decisions are made locally and distributively by the bidders themselves. So we could say that in reality, on-line ad-words auctions would always converge to a stable state. In this sense, the new concept is very robust.

6. ACKNOWLEDGEMENT

We would like to thank several anonymous referees whose suggestions have helped us to improve the presentation, and will also help us to significantly improve the writing in the finalized version for issues we have not been able to fully address in this revision due to limited time.

7. REFERENCES

[1] Iab internet advertising revenue report (2005 full-year). Available at `http://www.iab.net/resources/adrevenue/pdf/IAB_PwC_2005.pdf`, 2005.

[2] G. Aggarwal, A. Goel, and R. Motwani. Truthful auctions for pricing search keywords. In J. Feigenbaum, J. C.-I. Chuang, and D. M. Pennock, editors, *Proceedings 7th ACM Conference on Electronic Commerce (EC-06)*, pages 1–7, Ann Arbor, Michigan, USA, 11–15 2006. ACM.

[3] K. Asdemir. Dynamics of bidding in sponsored search auctions: An analytical investigation. In *Second Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-06)*, Ann Arbor, Michigan, June 11 2006.

[4] C. Borgs, J. Chayes, O. Etesami, N. Immorlica, K. Jain, and M. Mahdian. Bid optimization in online advertisement auctions. In *Second Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-06)*, Ann Arbor, Michigan, June 11 2006.

[5] F. Brandt and G. Weiß. Antisocial agents and vickrey auctions. In J.-J. Meyer and M. Tambe, editors, *Pre-proceedings of the Eighth International Workshop on Agent Theories, Architectures, and Languages (ATAL-2001)*, pages 120–132, Aug. 2001.

[6] X. Chen, X. Deng, and B. J. Liu. On incentive compatible competitive selection protocol. In *Lecture Notes in Computer Science*, volume 4112, pages 13–22, Taipei, Taiwan, Aug. 15 2006.

[7] E. H. Clarke. Multipart pricing of public goods. *Public Choice*, 11:11–33, 1971.

[8] B. Edelman and M. Ostrovsky. Strategic bidder behavior in sponsored search auctions. In *First Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-05)*, Vancouver, BC, Canada, June 5 2005.

[9] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second price auction: Selling billions of dollars worth of dollars worth of keywords. In *Second Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-06)*, Ann Arbor, Michigan, June 11 2006.

[10] T. Groves. Incentives in teams. *Econometrica*, 41:617–631, 1973.

[11] M.-Y. Kao, X.-Y. Li, and W. Wang. Output truthful versus input truthful: A new concept for algorithmic mechanism design, 2006.

[12] B. Kitts and B. Leblanc. Optimal bidding on keyword auctions. *Electronic Markets, Sepcial issue: Innovative Auction Markets*, 14(3):186–201, 2004.

[13] S. Lahaie. An analysis of alternative slot auction designs for sponsored search. In J. Feigenbaum, J. C.-I. Chuang, and D. M. Pennock, editors, *Proceedings 7th ACM Conference on Electronic Commerce (EC-06)*, pages 218–227, Ann Arbor, Michigan, USA, June 11–15 2006. ACM.

[14] C. Meek, D. M. Chickering, and D. B. Wilson. Stochastic and contingent-payment auctions. In *First Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-05)*, Vancouver, Canada, June 5 2005.

[15] H. R. Varian. Position auctions. Availble at `http://www.sims.berkeley.edu/~hal/Papers/2006/position.pdf`, Mar. 29 2006.

[16] W. Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, XVI:8–37, 1961.

[17] Y. Zhou and R. Lukose. Vindictive bidding in keyword auctions. In *Second Workshop on Sponsored Search Auctions, in conjunction with the ACM Conference on Electronic Commerce (EC-06)*, Ann Arbor, Michigan, June 11 2006.